Cyclic loading of shallow offshore foundations on sand

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ABSTRACT: A significant challenge currently in geotechnical engineering is the accurate modelling, both physical and theoretical, of cyclic loading. This is particularly important when considering the design of structures for the offshore environment, where much of the loading on the foundations is derived from the periodic wave action on the structure. An accurate understanding of how the foundation performs under cyclic loading is necessary so that over-conservatism in design can be reduced. Developing a rigorous understanding of such problems may have a significant impact on many applications in offshore engineering such as offshore wind farms, minimal facility offshore structures and mobile drilling units. This paper describes current developments in both experimental techniques and theory. The research is mainly aimed at suction caisson foundations but the general approach is equally applicable to other types of offshore and onshore shallow foundations subjected to cyclic loading.

1 INTRODUCTION

Physical modelling plays an important role in guiding how theoretical models might develop. This is particularly the case for cyclic loading, which although one of the more complex areas of soil mechanics, is nevertheless often treated empirically. Physical modelling of cyclic loading is most often carried out using sinusoidal wave loading. However, recently ‘pseudo-random’ loading sequences have been used in an attempt to reproduce real wave loads more accurately. The outcome of this research is that a strong relationship is established between the results of monotonic and cyclic loading, which provides the necessary information to guide the development of theoretical models.

This paper explores the relationship between cyclic and monotonic loading, illustrated by laboratory experiments carried out on model shallow foundations. The foundations are embedded in an oil-saturated sand so that drainage times are approximately comparable to a typical offshore situation. This ensures that a partially drained response can be obtained in the laboratory within the constraints of the loading apparatus. The link between cyclic and monotonic loading leads to a better understanding of the types of physical modelling that are most useful. Finally the paper will illustrate the use of a new type of theoretical modelling, named continuous hyperplasticity, which will demonstrate that an accurate and concise theory encapsulating cyclic loading is possible.

2 EXPERIMENTAL WORK

The principal aim of the experimental work was not to model any specific prototype situation but to guide the development of generic theoretical models that can be adapted to model prototype situations. Once the theoretical approaches are at an appropriate level of sophistication that they begin to replicate in a general sense the physical reality they can be fine tuned to particular site-specific problems. Issues of scaling can then be dealt with as appropriate. For example Bolton (1986) has illustrated the effect of stress level on the frictional response of sand. Although we acknowledge that there are issues of scaling for site-specific design there is no fundamental reason why in general the behaviour of the physical model should diverge significantly from the prototype. Earlier models, such as single surface plasticity models based on laboratory test results, have been used successfully for determining the extreme responses of jack-up rigs to harsh environmental conditions (see Cassidy, 1999).

The experiments were carried out at the University of Oxford. They were conducted at 1g using a specialised three degree-of-freedom loading device developed for exploring the response of foundations subjected to combined loading. The apparatus has been designed so that any of the three loads (vertical, moment or horizontal - $(V, M/2R, H)$) or displacements $(w, 2Rq, u)$ can be applied independently (assuming that the loading can be idealised as planar). Control algorithms have been implemented in the software so that specific load paths, which might correspond to wave loading, can be applied at rates of up to 1Hz. Loads and displacements are measured at the footing reference point, which is located at the centre of the underside of the footing (at mudline level). The soil samples used for the majority of testing were of a dense silica sand, at a relative density of 94%, saturated with a 100 centistoke silicon oil. The sample was 250mm deep and 1100mm in diameter. The oil is necessary so that drainage times comparable to typical offshore applications can be approximately modelled. Further details of the experimental set-up, including scaling of pore fluid, particle size and the actuating apparatus can be found in Byrne (2000).

The foundation used in the majority of the tests is a suction caisson. This is a relatively novel type of offshore foundation, that has, however, been used in several recent offshore applications. The caisson, shown in Figure 1, is essentially an upturned bucket that is installed by pumping out the water from the internal cavity, once the skirt has made sufficient contact with the seafloor. The bucket can for instance replace the piles at each corner of a jacket structure, or the drag anchors used in a floating facility mooring layout. The caisson is a preferred foundation option in many cases, as the installation cost can be significantly reduced. Typically the foundation might be installed within 24 hours, which compares to several weeks for piling (see Tjelta, 1994). The creation of the pressure differential when pumping out the water from the internal cavity has two effects for installation in a granular material. Firstly there is the net downward pressure over the foundation, which drives it into the ground. Secondly, and more importantly, the pressure differential sets up seepage gradients within the soil, which degrade the resistance at the skirt tip almost to zero (Erbrich and Tjelta, 1999). This enables the caisson to penetrate very dense sand deposits. In clays it is just the net downward force that drives the caisson into the ground.

To extend the application of caissons it is necessary to develop a rigorous analytical framework that enables a confident design under many different conditions. The most typical loading condition will be cyclic loading derived from the wave action on the structure. Design of shallow foundations for this loading condition is usually made on a case-specific basis (see Bye *et al.*, 1995) using empirically derived design guidelines, which may not be transferable to other sites. This paper examines some cyclic loading results which indicate a clear approach for the development of a rigorous design and modelling framework. The diameter of the footing used in the study was 150mm with a skirt depth of 50mm. The wall thickness was 0.45mm with all dimensions representing a 1:100 scaling of the Sleipner T platform bucket foundations (Erbrich and Tjelta, 1999).

As cyclic loading on the foundations is derived from the waves, which are typically pseudo-random, the testing employed a similar loading applied to model foundations. Such a loading history is shown in Figure 2. This load history, although appearing random, is deterministic in nature. The method, known as "Constrained NewWave", allows an extreme event to be inserted into an otherwise random loading sequence. This represents the cyclic wave loading in a more realistic manner than the usual sinusoidal loading. During the testing it was typical to have a number of smaller cycles before the large extreme event, so that the pore pressure regime around the foundation was similar to that which would occur in the field.

2.1 *Vertical Loading – Cyclic and Monotonic*

For many applications of caissons in granular materials the tensile response appears to be most critical. If the sand is very dense then the strongly dilatant behaviour of the sand, combined with the very fast loading rate relative to drainage times, ensures that a very high ultimate tensile capacity can be realised. This tensile capacity is limited by cavitation of the pore fluid within the soil matrix. Serviceability at lower tensions is, however, a concern, and load histories that are predominantly compressive but just reach tension are of particular interest. The load-displacement path for such a test is shown in Figure 3, where several load cycles just dip into tension. Clearly there is a major change in the vertical stiffness from a stiff to a much softer response once the load passes from compression to tension. These results indicate that, for dense sand, serviceability criteria may well dictate the design of the foundation rather than the ultimate capacity.

To investigate the softened tensile response further, the caisson was subjected to monotonic pull tests. In these the caisson is loaded to a vertical load (usually compressive), and then pulled into tension at a specified velocity. Such a test is shown in Figure 4, where the caisson was pulled up 2mm from a vertical load of 200N (the starting displacement was at the vertical axis). Superimposed on the graph are the extreme points of each cycle from a cyclic loading test carried out around a mean load of 200N. The two tests were at rates differing by two orders of magnitude. There is a high degree of similarity between the results, indicating that for small displacements monotonic and cyclic test results are related, and that there is little effect of loading rate. Other cyclic loading tests carried out at different rates also indicated that for small displacements loading rate had little effect on the response. However, at large displacements, particularly in tension, it is clear that the loading rate will affect the stiffness of the response, as it will determine the amount of pore fluid drainage. Further information about these results can be found in Byrne and Houlsby (2002a) and Byrne (2000).

2.2 *Combined Loading – Cyclic and Monotonic*

Combined cyclic loads were also applied to the footing. The cyclic loads were applied, for example, as moment loads whilst keeping the vertical load constant. The response of the footing depends on the previous stress history that the footing has undergone. If there has been little previous yielding of the footing there is likely to be considerable vertical displacement as plastic deformation occurs due to the cyclic moment loads. As for the vertical loading, there was little effect of rate in the experiments undertaken. There was also a close relationship between monotonic and cyclic results, in that a monotonic test forms a backbone curve through which the extreme points of the cycles pass.

Figure 5 shows three monotonic moment tests carried out at different mean vertical loads. Clearly as the mean vertical load is increased the stiffness of response also increases. Superimposed on the curves are extreme points from cyclic loading tests conducted at the same vertical loads. Each shows a very close relationship to the relevant monotonic test. This behaviour has been observed in many other studies of material response and implies that the hysteresis loops in cycles can be described by Masing's rules (Masing, 1926). The change in response at different mean vertical loads corresponds to a stiffness that varies with the square root of mean stress level, as has been observed previously for granular material (*e.g.* Wroth and Houlsby, 1985). Given that Masing behaviour can be used to describe the response, then new tests can be designed specifically to provide information for developments to the theory. Such a test is shown in Figure 6, where there are increasing cycles of load followed by decreasing cycles of load. Figure 7 shows the results from applying just increasing cycles of load to the footing. A clear backbone curve can be defined, which corresponds closely to the monotonic response. The reducing stiffness with cycle amplitude is clearly apparent. Further information about these results can be found in Byrne and Houlsby (2002b) and Byrne (2000).

3 THEORETICAL MODELLING

It is vital that experimental data are set in the context of an appropriate theoretical framework if the results are to be understood and applied to practical problems, and not merely used as collections of empirical data. This is particularly true in the case of cyclic loading, although this need is often neglected. We present here therefore an outline of an approach that can be used to model cyclic behaviour realistically, as the tests described above have been designed specifically to allow development of theoretical models within this framework.

Work hardening plasticity theory is a very successful approach for modelling the response of shallow foundations to monotonic loading. The concepts have their roots in the work of Roscoe and Schofield (1957), and have recently been extensively developed. Theoretical models for footing behaviour, based on experimental results, have for instance been developed by Martin (1994) for clay and Cassidy (1999) for sand. To develop such a model it is necessary to define (a) a yield surface which bounds the combinations of loading which would cause only elastic deformation, (b) a relationship for the hardening (enlargement) of the surface, (c) a flow rule specifying the relationship between plastic displacements once the load state reaches the yield surface, and, (d) the elastic behaviour within the yield surface. Such an approach has been found to be adequate for dense sands (Butterfield and Gottardi, 1994; Houlsby *et al.*, 1997), overconsolidated clays (Martin and Houlsby, 2001) and loose carbonate sands (Byrne and Houlsby, 2001). These models have reached the stage where they can be readily implemented within structural analysis packages so that detailed modelling of offshore structures can be carried out (see for example Cassidy, 1999).

Although the above models represent the current state-of-the-art, they are not able to replicate accurately the behaviour when repetitive loading, such as that shown in Figure 7, is applied to the foundation as they employ only a single yield surface. For example Figure 8 shows how a single surface plasticity model would model a test where increasing cycles of stress are applied to the footing. Initially the footing load state is located at the apex of yield surface. When a moment is applied to the footing the yield surface expands as plasticity occurs. On unloading a much stiffer response occurs as the footing load state is now entirely within the yield surface. On reloading, plasticity only occurs once the footing load state again reaches the yield surface. This clearly does not model well the behaviour shown in Figure 7.

An extension of these plasticity theories is to include multiple yield surfaces so that a gradual transition of stiffness can be approximated. Each surface must also be accompanied by a plastic potential to describe the direction of plastic flow. This type of theory has been developed for constitutive modelling of soils (see for example Houlsby, 1999). The main drawback is that a number of parameters must be specified for each yield surface. For accurate modelling of soil behaviour this can lead eventually to a rather unwieldy theory. More recently advances have been made in a theory termed 'continuous hyperplasticity'. A complete exposition of this theory is inappropriate here as it involves a considerable amount of mathematical development. Reference can be made to Collins and Houlsby (1997), Houlsby and Puzrin (2000) and especially Puzrin and Houlsby (2001a,b). This approach to plasticity theory has been formulated in such a way that any theory developed within it is guaranteed to obey thermodynamic principles.

In essence the theory replaces the 'plastic strain' in conventional plasticity theory with a continuous field of an infinite number of plastic strain components, each associated with a separate yield surface. This is achieved within a manageable mathematical framework by deriving the plasticity theory for a dissipative material entirely from two potentials. The first is the Gibbs free energy or the Helmholtz free energy. The second potential is the dissipation function. For the case of the infinite field of plastic strains these potentials are functionals ('functions of functions') of the plastic strain and its rate. Conventional plasticity theory is a special case of the new approach. The result is that theories can be constructed in which responses of the character shown in Figure 7 can be modelled accurately and with computational efficiency.

A mechanical system that reproduces such Masing (1926) behaviour is the spring and slider system (Iwan, 1967) shown in Figure 9. Puzrin and Houlsby (2001b) show that for such a system the Gibbs free energy can be defined as:

$$g = -\frac{P^2}{2k} + \int_0^1 \frac{k_1}{2}\hat{\alpha}^2\hat{\Gamma}\,d\eta - P\int_0^1 \hat{\alpha}\hat{\Gamma}\,d\eta \qquad (1)$$

where $\hat{\Gamma} = \Gamma(\eta)$ is a distribution function such that $\hat{\Gamma}d\eta$ is the fraction of the total number of yield surfaces having a dimensionless size parameter between η and $\eta + d\eta$. In this case the hardening stiffness k_1 is assumed constant, but the strengths of the sliding units are assumed to be proportional to η (*i.e.* each is $c\eta$). The dissipative stress for the system is given by:

$$d = \int_0^1 c\eta\left|\dot{\hat{\alpha}}\right|\hat{\Gamma}\,d\eta \qquad (2)$$

The dissipative generalised stress function, $\hat{\chi}$, and the field of yield functions are given by:

$$\hat{\chi} = \frac{\partial\hat{d}}{\partial\dot{\hat{\alpha}}} = c\eta\,\mathrm{sgn}\left(\dot{\hat{\alpha}}\right) \qquad (3)$$

$$\hat{y} = \hat{\chi} - |c\eta| = 0 \qquad (4)$$

Puzrin and Houlsby (2001b) show that the resulting incremental response is given by:

$$\dot{\delta} = \left[\frac{1}{k} + \frac{1}{k_1}\int_0^{\eta^*}\hat{\Gamma}\,d\eta\right]\dot{P} \qquad (5)$$

where η^* is the largest η such that $\hat{\chi} - |c\eta| = 0$. This model simulates the one-dimensional elastic non-linear plastic load displacement behaviour observed in the Iwan model as *N* (the number of blocks) tends to infinity. This type of behaviour fits well the experiments described above. The first term contributes an elastic displacement whilst the second is a plastic term depending on how many slider elements have been mobilised. Puzrin and Houlsby (2001b) show that the distribution function, $\hat{\Gamma}$, is related uniquely to the second derivative of the initial backbone curve, such as those which are shown in Figures 5 and 7. The relationship is:

$$\frac{d^2\delta}{dP^2} = \frac{1}{ck_1}\Gamma\left(\frac{P}{c}\right) \qquad (6)$$

The hyperbolic function that has been used to fit the backbone curves is:

$$\delta = \frac{P\left(c - \left(2 - k/k_{50}\right)P\right)}{k\left(c - P\right)} \qquad (7)$$

which leads to a distribution function of:

$$\hat{\Gamma} = \frac{k(1-\eta)^3}{2\left(k/k_{50} - 1\right)} \qquad (8)$$

where *k* is the initial stiffness, k_{50} is the stiffness at 50% stress and *c* is the peak strength. The formulation above has been defined in terms of a force-displacement pair (P, δ) which can easily take the form of (H, u) and $(M/2R, 2Rq)$. This formulation has been implemented in a numerical program and used to model some of the experimental work. For example Figure 7 shows the result of a moment test in which cycles of increasing amplitude have been applied. Figure 10 shows the fitted response using the continuous hyperplastic model. Whilst the fitting is not exact, the model captures the main features of the cyclic test.

4 CONCLUSIONS

This paper has presented some data from experiments conducted on shallow foundations embedded in saturated dense silica sand. The experiments included cyclic and monotonic loading tests. There was a strong correlation between the two different types of tests. Loading rate had a minimal effect on response. These observations allow a more focused testing program to be followed where tests are conducted to aid theoretical developments. A new theory was presented which could be used to reproduce the experimental data.

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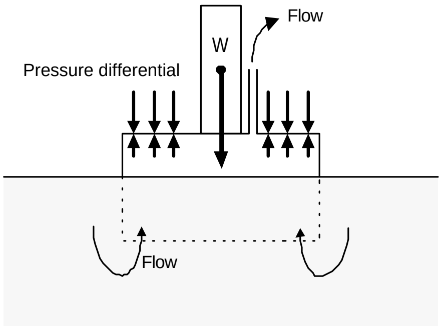

Figure 1. Diagram of the installation of a suction caisson.

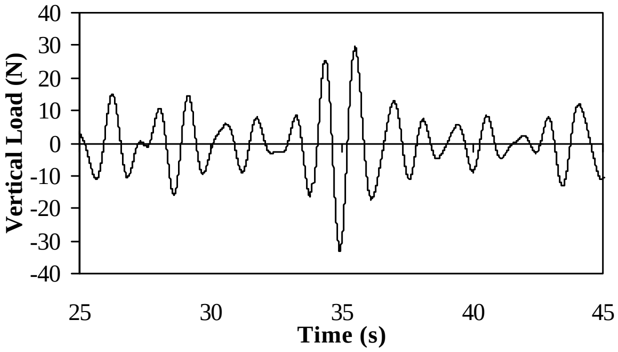

Figure 2. A typical psuedo-random cyclic load path.

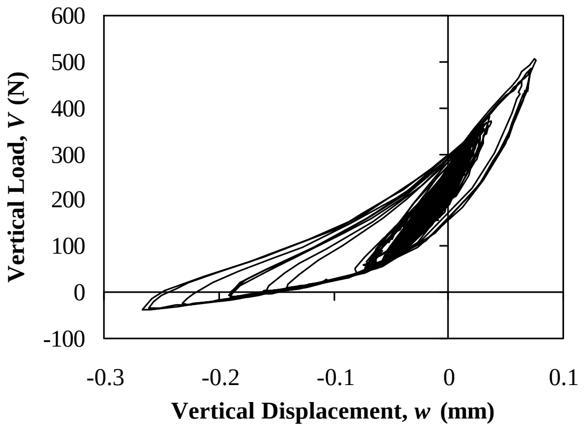

Figure 3. Vertical cyclic load-displacement response.

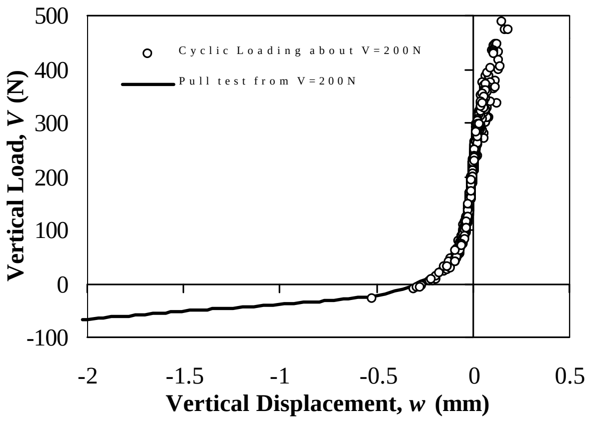

Figure 4. Vertical cyclic and monotonic tests.

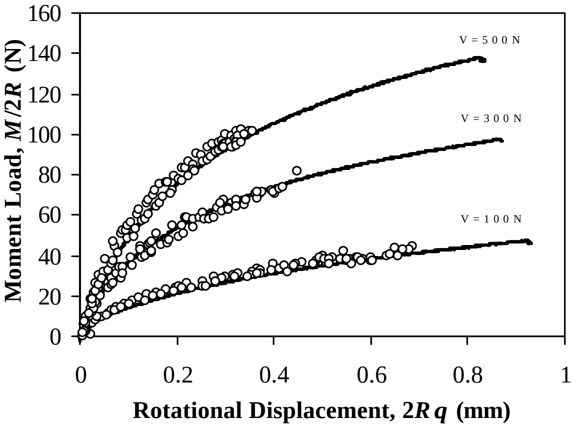

Figure 5. Moment cyclic and monotonic tests.

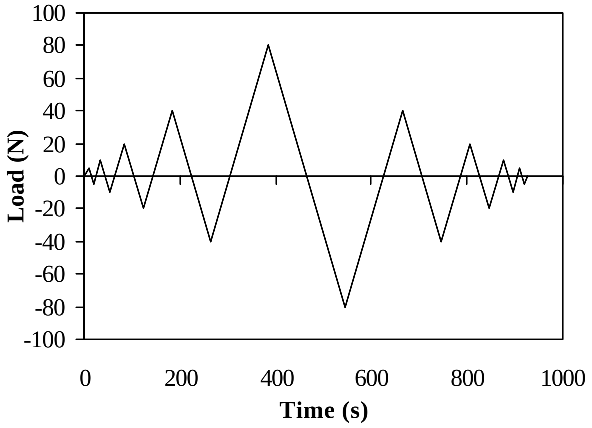

Figure 6. Optimal loading history for cyclic loading.

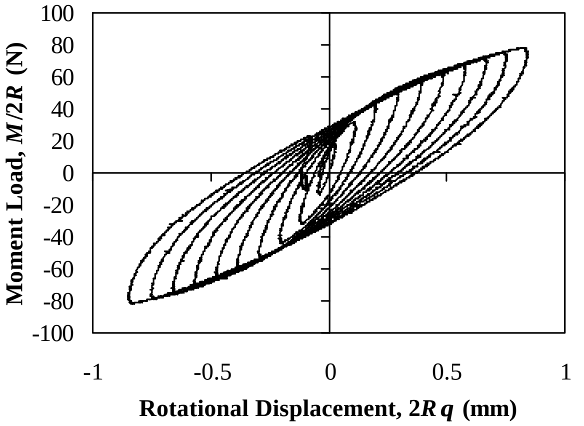

Figure 7. Response of footing to increasing cycles of stress.

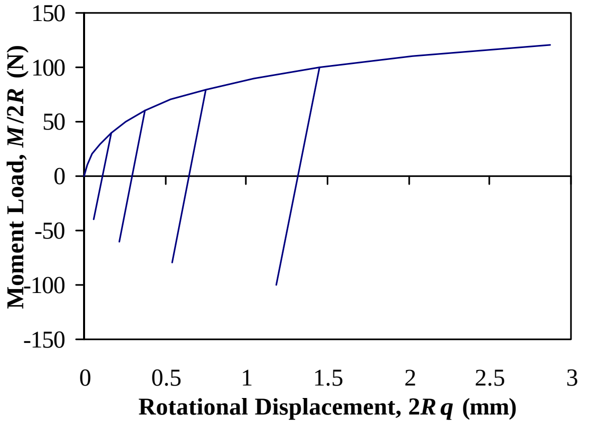

Figure 8. Current theoretical modelling of increasing cycles.

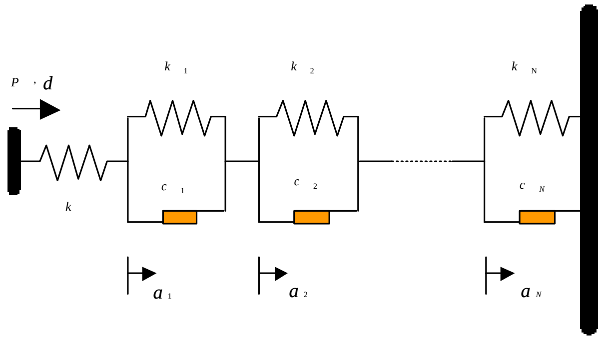

Figure 9. Spring and slider system after Iwan (1967).

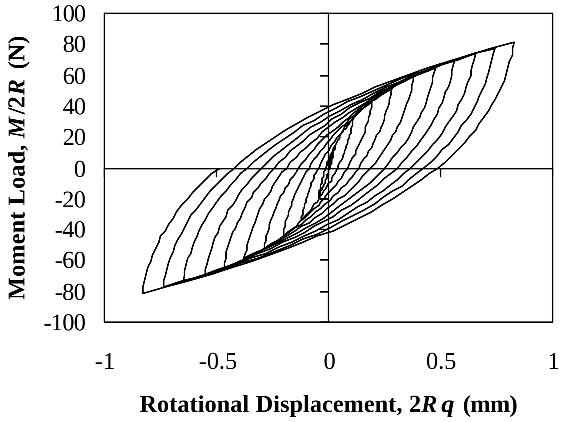

Figure 10. ‘Continuous hyperplasticity’ modelling of response.