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TECHNICAL NOTE

Vertical bearing capacity factors for conical footings on sand

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KEYWORDS: bearing capacity; numerical modelling and analysis; offshore engineering; sands

INTRODUCTION

The drained bearing capacity of a circular foundation on the surface of frictional material, loaded by a central vertical load, is usually calculated using the well-known procedures described by Terzaghi (1943) and formulated as

$$V = \gamma' N_\gamma \pi R^3 \tag{1}$$

where R is the footing radius and γ' is the effective unit weight of the soil. Various authors have proposed appropriate values for the dimensionless bearing capacity, N_γ. However, most values are calculated for strip footings, and are applied to circular foundations by scaling using an empirical shape factor (usually taken as 0·6).

The most common procedure for calculating the bearing capacity factors theoretically has become the method of characteristics. For instance, Bolton & Lau (1993) calculated N_γ values directly for circular footings using this method for both rough and smooth footings. However, their solutions are limited to flat footings and cannot be applied to conical footings. The principal alternative approach is finite element analysis, but it is notoriously difficult to obtain accurate solutions to problems involving high angles of friction by this method.

This note presents a comprehensive set of bearing capacity factors for foundations of various cone angles, a range of roughnesses and angles of friction. The method used involves application of the method of characteristics as described by Houlsby (1982) and Houlsby & Wroth (1982). Previously, the method of characteristics has been applied to obtain axisymmetric solutions for both frictional and frictionless soils by Cox *et al.* (1961) and Cox (1962), and for sands by Bolton & Lau (1993). Furthermore, while investigating the applicability of the fall cone test to determine the liquid limit of clays, Houlsby (1982) determined the vertical bearing capacity of a limited number of cones on clay. Martin (1994) extended this analysis to 1296 combinations of cone angle, footing embedment, roughness factor and increases of undrained shear strength with depth. However, no thorough analysis of the bearing capacity of conical footings on sand has been published.

The motivation for this numerical study was the calculation of the vertical (pre-load) penetration of the spudcan foundations of jack-up units used for offshore drilling. These are typically large shallow conical footings (Cassidy, 1999; Cassidy & Houlsby, 1999). Whereas the detailed geometry of spudcans often involves a step in the cone angle (usually, but not always, with a sharper tip), they can be approximated with reasonable accuracy as cones of constant angle. This study follows the publication of Martin's (1994) set of bearing capacity factors for conical footings on clay in the current industry standard for jack-up assessment (SNAME, 1997). However, the bearing capacity factors derived in this study could be applied to any conical footing on sand. Values of cone angle outside the normal range for spudcans are also included.

SOLUTION METHOD

To derive bearing capacity values, collapse loads were calculated for this axisymmetric problem using the method of characteristics, implemented numerically in the program FIELDS. The theoretical methods used in FIELDS, and a sample numerical study of conical footings on clay, can be found in Houlsby (1982) and Houlsby & Wroth (1983). The analysis makes use of the lower bound theorem of plasticity theory. Therefore the solution satisfies the boundary conditions, the equilibrium equations and the yield criterion of the soil. In FIELDS the equations of equilibrium are combined directly with the yield condition to give a set of differential equations that can be solved throughout the region of interest (Houlsby, 1982). Within the study performed here, the sand was assumed to be rigid-plastic and to obey the Mohr–Coulomb yield criterion. Furthermore, as consistent with the bound theorems, the effects of change in geometry were not taken into account. For a penetrating cone (a spudcan for instance) the factors derived here represent the load at a particular penetration of the footing. Heave around a penetrating cone due to displaced material may be expected, but this has not been considered here. The effect of heave would be to increase the capacity slightly (Houlsby, 1982). The friction angle of the soil is taken as constant.

Formally a lower bound is established rigorously only if two further conditions are satisfied. First, it must be possible to extend the calculated stress field throughout the soil without violating the equilibrium or yield conditions. Although this extension has not been carried out as part of this study, experience of this type of relatively well-conditioned problem indicates that such an extension is always possible in these cases. Second, the bound theorems themselves are proven only for materials with 'associated' flow rules. For this to be the case the angle of dilation would have to be equal to the angle of friction, whereas in reality it is much lower. This means that these solutions cannot be regarded as rigorous lower bounds, but nevertheless there is broad acceptance that equilibrium solutions using the angle of friction are practically useful solutions.

PROBLEM DEFINITION

Figure 1 defines the problem and notation. The geometry of the conical footing is described by the cone apex angle, β, and the radius of the footing, R. The surface roughness is specified by a roughness factor, α, such that the interface friction angle, δ, is given by $\tan\delta = \alpha \tan\phi$. A roughness

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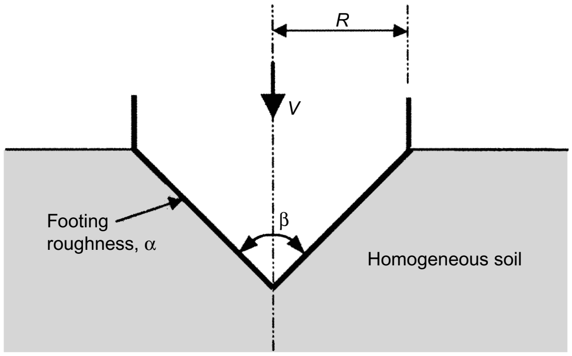

Fig. 1. Problem definition and notation

factor of $\alpha = 0$ represents a smooth footing, whereas $\alpha = 1{\cdot}0$ represents a fully rough one. The vertical force exerted by the cone on the soil is V and the properties of the sand are given by the constant friction angle, ϕ, and the effective unit weight, γ'. For a homogeneous soil, the method of characteristics has been applied to find a lower bound value of V, and this is then normalised by γ' and R to evaluate a non-dimensional N_γ by a rearrangement of equation (1):

$$N_\gamma = \frac{V}{\gamma' \pi R^3} \tag{2}$$

All 360 combinations of the following three dimensionless parameters were investigated:

(*a*) cone apex angle, $\beta = 30°, 60°, 90°, 120°, 150°, 180°$
(*b*) roughness factor, $\alpha = 0, 0{\cdot}2, 0{\cdot}4, 0{\cdot}6, 0{\cdot}8, 1{\cdot}0$
(*c*) friction angle, $\phi = 5°, 10°, 15°, 20°, 25°, 30°, 35°, 40°, 45°, 50°$.

All of the factors were derived for a general shear failure mechanism. A typical example generated by the program FIELDS is illustrated in Fig. 2 (for this case $\beta = 90°$, $\alpha = 0{\cdot}2$ and $\phi = 20°$). This is as expected for shallow foundations, with the field of slip lines extending from the underside of the footing to the horizontal free surface. The stress field has not been extended throughout the entire soil region, as indicated in Fig. 2, and therefore the solution is not a rigorous lower bound. However, the region shown in Fig. 2 is sufficient to obtain an accurate vertical load value (Houlsby, 1982).

Lower bound bearing capacity factors for all combinations of the dimensionless parameters are shown in Fig. 3, and numerical values are given in Tables 1–6. It can be seen that the strongest influence on the bearing capacity factor is, as expected, the angle of friction. The cone roughness has a secondary influence, and is proportionately more important for sharper cones and higher angles of friction. Finally the cone angle has a slightly more complex effect. In general the higher the cone angle the lower the bearing capacity factor: this is because the sharper cones (small cone angles) mobilise the strength of deeper material at a higher stress level. However, for high angles of friction a counteracting trend occurs because high stress gradients can be sustained through the ‘fan zone’ at the edge of the cone, which is larger for the higher cone angles. The result is that for high angles of friction a minimum bearing capacity occurs at some intermediate cone angle, but the position of the minimum varies with both angle of friction and roughness: see Fig. 4.

DISCUSSION

In Fig. 5 the lower bound solutions for the flat plate ($\beta = 180°$) derived in this study have been compared with those suggested by Bolton & Lau (1993). The full set of friction angles are shown, but only for the smooth and rough cases. Almost exact agreement can be seen for the smooth footings. However, this is not the case for the rough footings. The values in this study are significantly smaller than those suggested by Bolton & Lau, especially for the lower friction angles. In the calculations presented here the shape of the trapped zone of soil beneath the centre of the footing has been calculated according to the procedure used by Davis & Booker (1973). This procedure ensures that the calculated bearing capacity is as high as possible, while satisfying the equilibrium and yield conditions. Bolton & Lau follow Meyerhof (1951) in assuming that there is a conical region with an apex angle of ϕ that is trapped beneath the footing, and that full friction is mobilised on the outside of this cone. This procedure has some justification when applied to N_q calculations for rough strip footings, but must be used with caution when applied to N_γ calculations for circular footings. A check should have been made that the stress field can indeed be extended within the trapped cone of soil without violating yield. We believe that the higher capacity values calculated by Bolton & Lau are an indication that such an extension is not indeed possible, and that their solutions therefore represent a slight overestimate of the bearing capacity.

Figure 5 also shows a comparison between the values for a flat plate suggested in this study and the current proposals for Eurocode 7, which effectively use the formula for a circular footing:

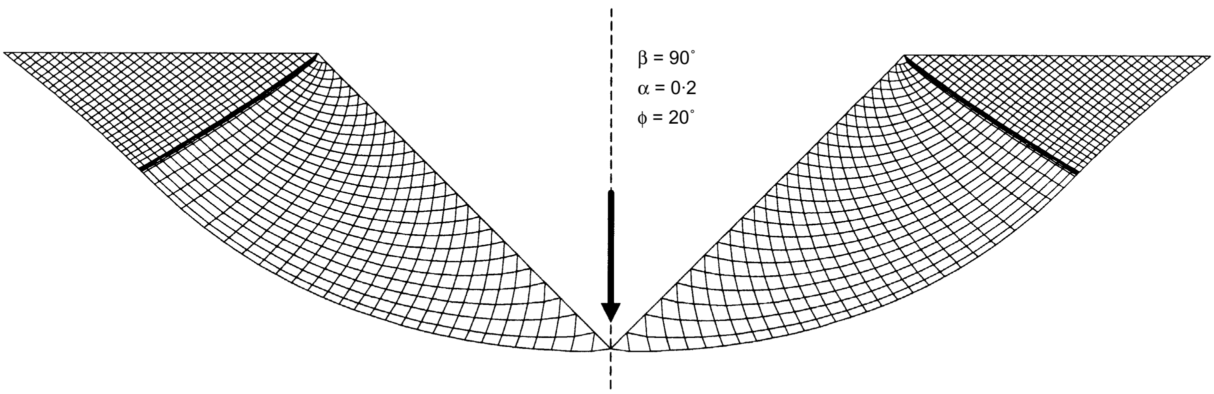

Fig. 2. Example general shear failure mechanism generated by program FIELDS

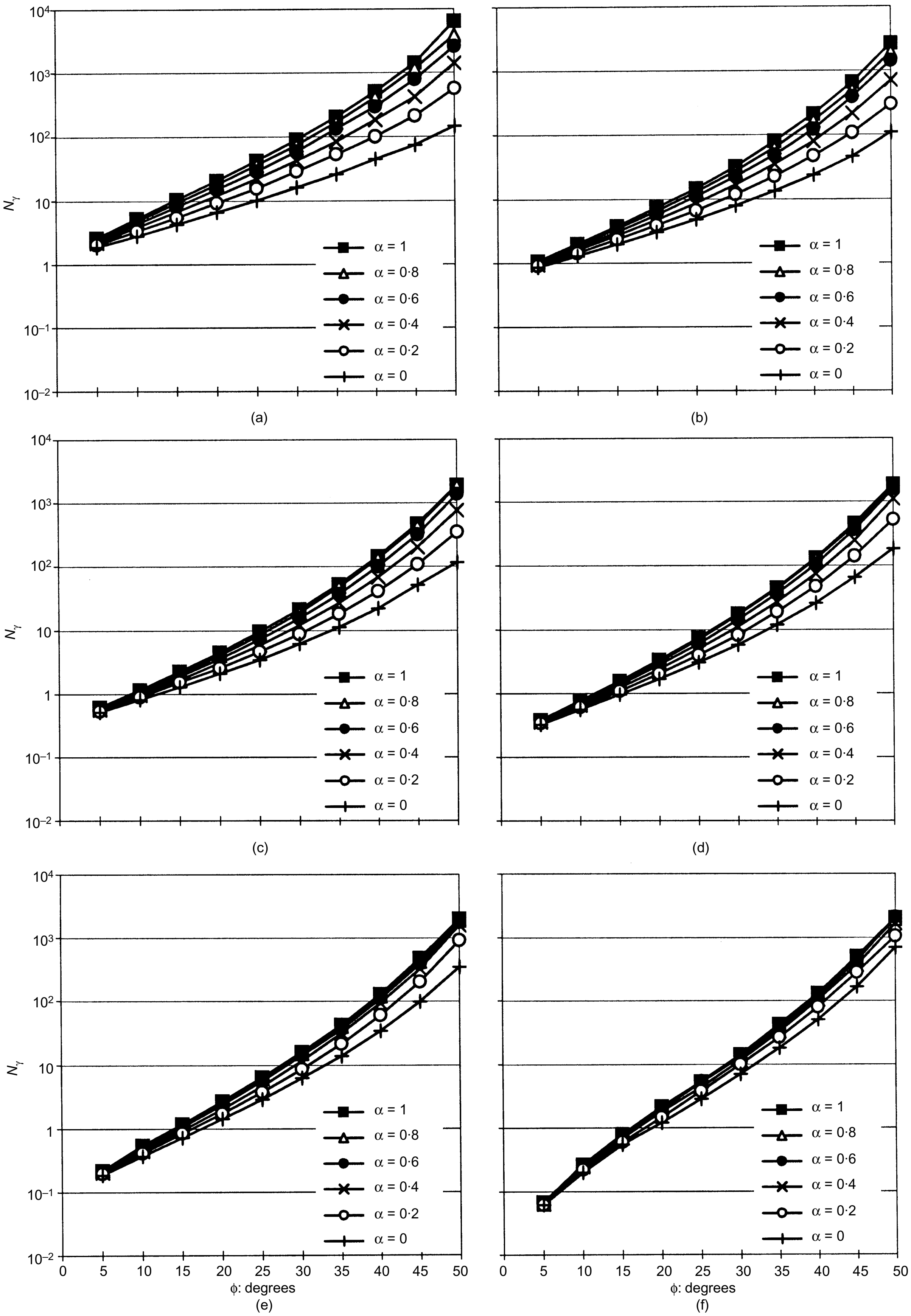

Fig. 3. Bearing capacity factors, N_γ: (a) $\beta = 30°$; (b) $\beta = 60°$; (c) $\beta = 90°$; (d) $\beta = 120°$; (e) $\beta = 150°$; (f) $\beta = 180°$

Table 1. Bearing capacity factors (N_γ) for a conical apex angle of 30°

ϕ: degrees	$\alpha = 1$	$\alpha = 0{\cdot}8$	$\alpha = 0{\cdot}6$	$\alpha = 0{\cdot}4$	$\alpha = 0{\cdot}2$	$\alpha = 0$
5	2·626	2·483	2·333	2·183	2·028	1·870
10	5·290	4·821	4·324	3·823	3·318	2·825
15	10·49	9·307	7·987	6·712	5·482	4·294
20	20·86	18·08	15·10	12·09	9·232	6·619
25	42·39	36·07	29·08	22·18	15·70	10·15
30	89·80	75·22	58·92	42·45	29·05	16·26
35	203·4	166·2	132·8	84·73	53·14	25·82
40	504·1	410·7	295·4	183·7	101·3	45·24
45	1420·3	1149·1	787·5	417·3	211·7	74·34
50	6504·3	3871·1	2567·7	1388·2	562·1	145·0

Table 2. Bearing capacity factors (N_γ) for a conical apex angle of 60°

ϕ: degrees	$\alpha = 1$	$\alpha = 0{\cdot}8$	$\alpha = 0{\cdot}6$	$\alpha = 0{\cdot}4$	$\alpha = 0{\cdot}2$	$\alpha = 0$
5	1·073	1·036	0·995	0·953	0·909	0·863
10	1·999	1·872	1·735	1·591	1·446	1·299
15	3·775	3·445	3·080	2·708	2·340	1·979
20	7·333	6·546	5·639	4·752	3·896	3·075
25	14·69	12·99	10·94	8·660	6·639	4·820
30	31·99	27·45	22·50	16·70	11·86	7·950
35	79·26	62·95	48·81	34·83	22·57	13·36
40	209·2	163·2	122·3	80·13	47·27	23·75
45	646·3	495·8	382·4	214·1	108·0	46·05
50	2650·0	1913·2	1414·7	698·1	295·3	108·8

Table 3. Bearing capacity factors (N_γ) for a conical apex angle of 90°

ϕ: degrees	$\alpha = 1$	$\alpha = 0{\cdot}8$	$\alpha = 0{\cdot}6$	$\alpha = 0{\cdot}4$	$\alpha = 0{\cdot}2$	$\alpha = 0$
5	0·625	0·605	0·586	0·565	0·544	0·521
10	1·175	1·113	1·044	0·972	0·896	0·818
15	2·269	2·100	1·909	1·709	1·540	1·313
20	4·540	4·114	3·625	3·114	2·600	2·115
25	9·581	8·502	7·257	5·936	4·663	3·509
30	21·12	18·87	15·58	12·09	8·911	6·219
35	51·76	47·42	37·01	26·95	18·26	11·26
40	142·8	132·6	99·18	67·48	40·80	22·13
45	458·7	419·5	312·6	199·6	107·1	51·30
50	1923·3	1850·4	1384·0	751·8	340·1	115·2

Table 4. Bearing capacity factors (N_γ) for a conical apex angle of 120°

ϕ: degrees	$\alpha = 1$	$\alpha = 0{\cdot}8$	$\alpha = 0{\cdot}6$	$\alpha = 0{\cdot}4$	$\alpha = 0{\cdot}2$	$\alpha = 0$
5	0·386	0·379	0·368	0·356	0·343	0·329
10	0·778	0·740	0·708	0·660	0·611	0·561
15	1·595	1·510	1·387	1·248	1·105	0·963
20	3·373	3·155	2·810	2·437	2·055	1·688
25	7·460	6·987	6·046	5·008	3·994	3·064
30	17·58	16·80	14·30	11·04	8·232	5·768
35	44·73	42·99	35·79	26·74	18·67	11·88
40	129·4	124·9	103·3	73·810	46·44	25·84
45	446·8	410·0	359·1	244·2	138·4	65·00
50	1905·4	1840·3	1580·0	1089·4	518·4	180·2

$$N_\gamma = 0{\cdot}7\left[\mathrm{e}^{\pi \tan\phi}\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)-1\right]\tan\phi \qquad (3)$$

The Eurocode calculation falls close to the smooth footing calculation for both very low and very high friction angles, but in between it gives a higher value. Up to about $\phi = 30°$ it gives a higher figure than our rough calculations would suggest, although lower than Bolton & Lau's. Above this friction angle it falls between our rough and smooth calculations.

The Eurocode calculation is stated to be valid for '$\delta \geqslant \phi/2$ (rough base)', where δ is the interface friction angle on the base. In fact the Eurocode calculation does not follow our figures particularly closely for a rough footing, and it is particularly worrying that at friction angles below $\phi = 30°$ it exceeds our figures for a rough base (by a factor

Table 5. Bearing capacity factors (N_γ) for a conical apex angle of 150°

ϕ: degrees	$\alpha = 1$	$\alpha = 0{\cdot}8$	$\alpha = 0{\cdot}6$	$\alpha = 0{\cdot}4$	$\alpha = 0{\cdot}2$	$\alpha = 0$
5	0·221	0·220	0·211	0·206	0·198	0·188
10	0·548	0·503	0·478	0·442	0·411	0·374
15	1·189	1·140	1·059	0·951	0·843	0·728
20	2·725	2·609	2·386	2·055	1·752	1·428
25	6·441	6·126	5·604	4·632	3·774	2·890
30	15·93	15·13	14·02	11·64	8·632	6·270
35	42·36	40·42	36·41	31·34	21·53	13·79
40	128·1	120·5	110·5	94·07	60·86	34·36
45	477·1	421·6	387·7	328·2	201·6	99·10
50	1981·3	1881·9	1746·5	1584·3	916·6	347·2

Table 6. Bearing capacity factors (N_γ) for a conical apex angle of 180°

ϕ: degrees	$\alpha = 1$	$\alpha = 0{\cdot}8$	$\alpha = 0{\cdot}6$	$\alpha = 0{\cdot}4$	$\alpha = 0{\cdot}2$	$\alpha = 0$
5	0·067	0·064	0·064	0·063	0·062	0·062
10	0·266	0·261	0·264	0·246	0·223	0·200
15	0·796	0·787	0·751	0·691	0·601	0·553
20	2·160	2·041	1·990	1·802	1·477	1·219
25	5·270	5·375	5·144	4·371	3·776	2·865
30	14·13	13·91	12·98	11·38	9·891	6·935
35	42·56	40·93	36·81	32·69	26·61	17·88
40	129·4	121·5	117·0	103·0	79·44	50·46
45	505·0	459·4	427·9	363·5	279·3	165·1
50	2050·0	1920·0	2079·6	1656·6	1050·2	703·1

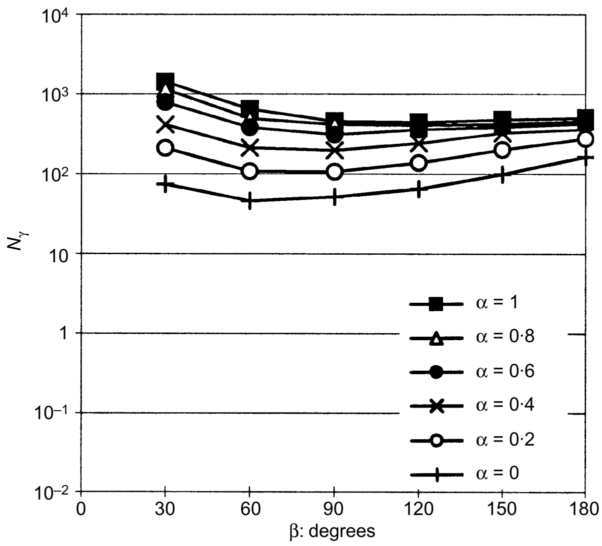

Fig. 4. Variation of N_γ with cone angle, β, for $\phi = 45°$

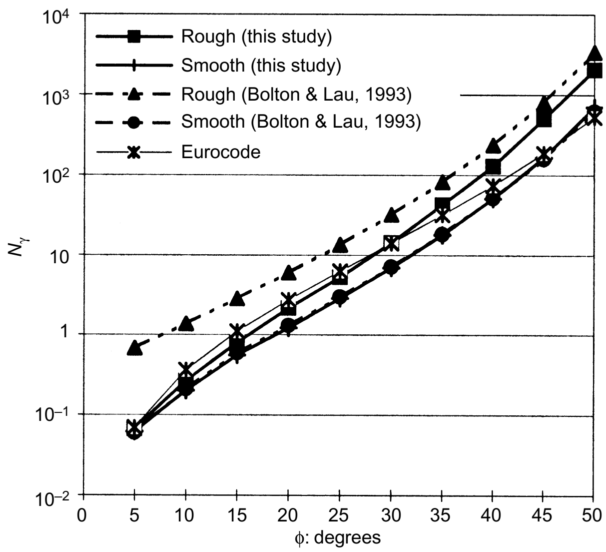

Fig. 5. Comparison of N_γ for $\beta = 180°$ with Bolton & Lau (1993) for flat footings

as high as 1·39 for $\phi = 15°$). We believe our figures to be as high as can be justified on the basis of a rational engineering calculation. We note that the Eurocode figures are based on:

(*a*) the exact solution for plane strain, $N_q = e^{\pi \tan \phi} \tan^2 (\pi/4 + \phi/2)$
(*b*) the empirical approximate calculation for plane strain, $N_\gamma = (N_q - 1) \tan \phi$
(*c*) a shape factor for converting to a circular footing of 0·7.

In conclusion we believe there is a case for modification of the formulae in Eurocode 7 for N_γ for circular foundations.

CONCLUSION

A comprehensive series of drained vertical bearing capacity factors have been presented for conical footings on homogeneous sand. Covering the full practical range of cone apex angles (from 30° to a flat footing), roughness (from smooth to fully rough) and friction angles (5° to 50°), the bearing capacity factors presented here represent all those necessary for practical applications. When applied to spudcans on dense sand a roughness factor close to 1 is appropriate, and the friction angle would typically lie in the range 35–50°. Comparisons with published solutions for the flat plate have been made.

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NOTATION

α roughness factor
β cone apex angle
γ' unit weight of soil
δ interface friction angle
N_γ dimensionless bearing capacity factor
ϕ friction angle
R footing radius
V vertical bearing capacity

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